CS 61A
Spring 2026

Scheme, Scheme Lists

Discussion 9: April 8, 2026

Attendance

Your TA will come around during discussion to check you in. You can start on the worksheet before being checked in; you don't need to wait for your TA to get started.

Scheme

You can use scheme.cs61a.org to try things out.

An `if` expression has 3 subexpressions:

- The first one, called the *predicate*, is always evaluated to choose between the next two.
- The second one, called the *consequent*, is evaluated if the *predicate*'s value is true.
- The third one, called the *alternative*, is evaluated if the *predicate*'s value is false.

The value of the second or third expression (whichever gets evaluated) is the value of the whole `if` expression.

Only `#f` is a false value in Scheme. All other values are true values, including `#t`.

The logical special forms `and` and `or` can have any number of sub-expressions. Use `and` to check if all its sub-expressions are true values. Use `or` to check if any of them are.

An Example:

```
scm> (define y (+ 1 2))  ; y is now 3
y
scm> (define z (or (> y 4) (> y 5) (< y 5)))  ; z is true because y < 5
z
scm> (+ 7 (if (and (> y 1) z) y 1))  ; the if expression evaluates to 3
10
```

The `expect` form is built into scheme.cs61a.org and checks to see if its first sub expression evaluates to its second one. It is used for testing.

Q1: Perfect Fit

Definition: A perfect square is `k*k` for some integer `k`.

Implement `fit`, which takes non-negative integers `total` and `n`. It returns whether there are `n` **different** positive perfect squares that sum to `total`.

Use the `(or _ _)` special form to combine two recursive calls: one that uses `k*k` in the sum and one that does not.

```
;;; Return whether there are n perfect squares with no repeats that sum to total
;;;
;;; scm> (fit 10 2)
;;; #t
;;; scm> (fit 9 2)
;;; #f
(define (fit total n)
    (define (f total n k)
        (if (and (= n 0) (= total 0))
            #t
        (if (< total (* k k))
            #f
            'YOUR-CODE-HERE



        )))
    (f total n 1))

(expect (fit 10 2) #t)  ; 1*1 + 3*3
(expect (fit 9 1)  #t)  ; 3*3
(expect (fit 9 2)  #f)  ;
(expect (fit 9 3)  #f)  ; 1*1 + 2*2 + 2*2 doesn't count because of repeated 2*2
(expect (fit 25 1)  #t) ; 5*5
(expect (fit 25 2)  #t) ; 3*3 + 4*4
```

Scheme Lists & Quotation

Scheme lists are linked lists. Lightning review:

- `nil` and `()` are the same thing: the empty list.
- `(cons first rest)` constructs a linked list with `first` as its first element. and `rest` as the rest of the list, which should always be a list.
- `(car s)` returns the first element of the list `s`.
- `(cdr s)` returns the rest of the list `s`.
- `(list ...)` takes n arguments and returns a list of length n with those arguments as elements.
- `(append ...)` takes n lists as arguments and returns a list of all of the elements of those lists.
- `(draw s)` draws the linked list structure of a list `s`. It only works on code.cs61a.org/scheme. **Try it now with something like `(draw (cons 1 nil))`.**

Quoting an expression leaves it unevaluated. Examples:

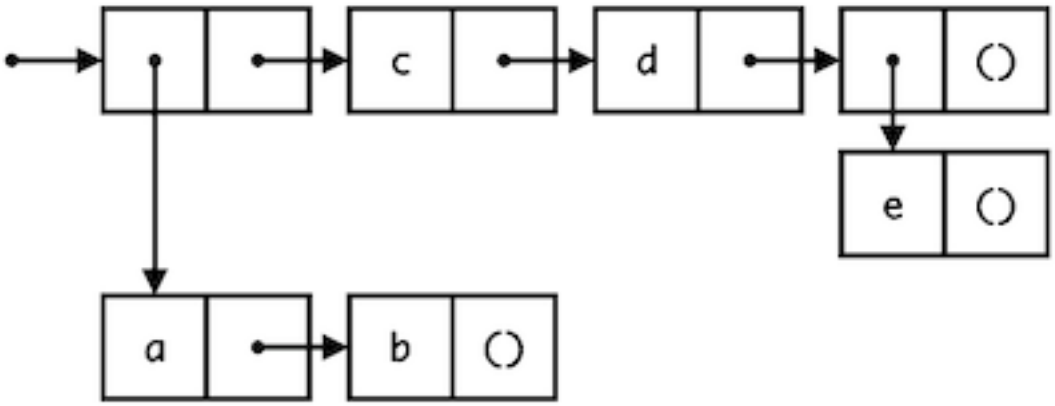
- `'four` and `(quote four)` both evaluate to the symbol `four`.
- `'(2 3 4)` and `(quote (2 3 4))` both evaluate to a list containing three elements: 2, 3, and 4.
- `'(2 3 four)` and `(quote (2 3 four))` evaluate to a list containing 2, 3, and the symbol `four`.

Here's an important difference between `list` and quotation:

```
scm> (list 2 (+ 3 4))
(2 7)
scm> '(2 (+ 3 4))
(2 (+ 3 4))
```

Q2: Nested Lists

Create the nested list depicted below three different ways: using `list`, `quote`, and `cons`.



Next, use calls to `list` to construct this list. If you run this code and then `(draw with-list)` in code.cs61a.org, the `draw` procedure will draw what you've built.

```
(define with-list
    (list
        'YOUR-CODE-HERE



    )
)
; (draw with-list)  ; Uncomment this line to draw with-list
```

Now, use `quote` to construct this list.

```
(define with-quote
    '(
        'YOUR-CODE-HERE



    )

)
; (draw with-quote)  ; Uncomment this line to draw with-quote
```

Now, use `cons` to construct this list. Don't use `list`. You can use `first` in your answer.

```
(define first
    (cons 'a (cons 'b nil)))
```

```
(define with-cons
    (cons
        'YOUR-CODE-HERE



    )
)
; (draw with-cons)  ; Uncomment this line to draw with-cons
```

Q3: Remove

Implement a procedure `remove` that takes in a list of numbers `s` and a number `x`. It returns a list with all instances of `x` removed from `s`. You may assume that `s` only contains of numbers and will not have nested lists.

```
;;; Return a list with all numbers equal to x removed
;;;
;;; scm> (remove '(3 4 3 4 4 3) 3)
;;; (4 4 4)
;;; scm> (remove '(3 4 3 4 4 3) 4)
;;; (3 3 3)
(define (remove s x)
  (if (null? s) nil
    (if (equal? x ____) ____ ____)
))

(expect (remove '(3 4 3 4 4 3) 3) (4 4 4))
(expect (remove '(3 4 3 4 4 3) 4) (3 3 3))
```

Q4: Pair Up

Implement `pair-up`, which takes a list `s`. It returns a list of lists that together contain all of the elements of `s` in order. Each list in the result should have 2 elements. The last one can have up to 3.

Look at the examples together to make sure everyone understands what this procedure does.

```
;;; Return a list of pairs containing the elements of s.
;;;
;;; scm> (pair-up '(3 4 5 6 7 8))
;;; ((3 4) (5 6) (7 8))
;;; scm> (pair-up '(3 4 5 6 7 8 9))
;;; ((3 4) (5 6) (7 8 9))
(define (pair-up s)
    (if (<= (length s) 3)
        'YOUR-CODE-HERE




    ))

(expect (pair-up '(3 4 5 6 7 8)) ((3 4) (5 6) (7 8)) )
(expect (pair-up '(3 4 5 6 7 8 9)) ((3 4) (5 6) (7 8 9)) )
```

Q5: Increasing Rope

Definition: A *rope* in Scheme is a non-empty list containing only numbers except for the last element, which may either be a number or a rope.

Implement `up`, a Scheme procedure that takes a positive integer `n`. It returns a rope containing the digits of `n` that is the shortest rope in which each pair of adjacent numbers in the same list are in increasing order.

Reminder: the `quotient` procedure performs floor division, like `//` in Python. The `remainder` procedure is like `%` in Python.

```
(define (up n)
    (define (helper n result)
        (define first (remainder n 10))  ; Using first will shorten your code
        (if (zero? n) result
            (helper
                (quotient n 10)
                'YOUR-CODE-HERE




                )))
    (helper
      (quotient n 10)
      'YOUR-CODE-HERE


    ))

(expect (up 314152667899) (3 (1 4 (1 5 (2 6 (6 7 8 9 (9)))))))
```

Tail Context

When trying to identify whether a given function call within the body of a function is a tail call, we look for whether the call expression is in **tail context**.

Given that each of the following expressions is the last expression in the body of the function, the following expressions are tail contexts:

1. the second or third operand in an `if` expression
2. any of the non-predicate sub-expressions in a `cond` expression (i.e. the second expression of each clause)
3. the last operand in an `and` or an `or` expression
4. the last operand in a `begin` expression's body
5. the last operand in a `let` expression's body

For example, in the expression `(begin (+ 2 3) (- 2 3) (* 2 3))`, `(* 2 3)` is a tail call because it is the last operand expression to be evaluated.

Q6: Is Tail Call

For each of the following procedures, identify whether it contains a recursive call in a tail context. Also indicate if it uses a constant number of active frames.

```
(define (question-a x)
  (if (= x 0) 0
      (+ x (question-a (- x 1)))))
```

```
(define (question-b x y)
  (if (= x 0) y
      (question-b (- x 1) (+ y x))))
```

```
(define (question-c x y)
  (if (> x y)
      (question-c (- y 1) x)
      (question-c (+ x 10) y)))
```

```
(define (question-d n)
  (if (question-d n)
      (question-d (- n 1))
      (question-d (+ n 10))))
```

```
(define (question-e n)
  (cond ((<= n 1) 1)
        ((question-e (- n 1)) (question-e (- n 2)))
        (else (begin (print 2) (question-e (- n 3))))))
```